Tiling a Chessboard:
A Problem Adapted for the Virtual Math Circle

● ● ●

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Outline

- Tucson Math Circle

- Anatomy of Miro (free collaborative whiteboard)

- Tiling Activities

- (Brief) Anatomy of GeoGebra

- Variations

Tucson Math Circle

Remote online format

- Started Fall 2020

  Christina Durón
  Nicole Sullivant
  Doug Pfeffer

- Zoom and Miro.com

Tucson Math Circle

Remote online format

- Started Fall 2020
  - Christina Durón
  - Nicole Sullivant
  - Doug Pfeffer

- Zoom and Miro.com
- Two student groups: Junior (K-6) and Senior (7-12)
  - Christina Durón
  - John Peca-Medlin
  - Bruce Bayly
  - Megan Lewis

- **Our goal**: To have students explore and enjoy topics in mathematics in a way that is not part of the standard K-12 curriculum

Tucson Math Circle

Challenges

1. Wide range of ages/grades
   - ➢ Cannot expect learning/recall of technical details
2. Attendance is flexible
   - ➢ Students may miss weeks
3. Student fatigue during the scheduled meetings
   - ➢ Wednesday late afternoon
   - ➢ One hour per meeting
   - ➢ Biweekly meetings

Our Solutions

1. Incorporate warm-ups

2. Focus on demonstrations

3. Rely on group discussions and interactive activities

Anatomy of Miro

tinyurl.com/JMM-Tiling-Problem

Anatomy of Miro

Anatomy of Miro

SCAN ME

Student A

$8 \times 8 = 64$

$64 \div 2 = 32$

32

Thickness

Anatomy of Miro

Movable

Anatomy of Miro

Rotatable

Anatomy of Miro

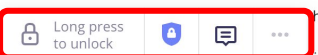Rotatable

Anatomy of Miro

Directions: Start with an uncovered chessboard. Take one of the "off limits" markers (X) and use it to cover two spaces.

Following Rule 1, Rule 2, and Rule 3, which boards can be covered?
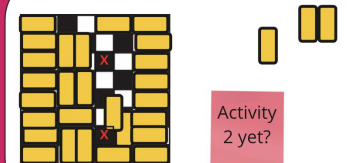
Which boards cannot be covered?

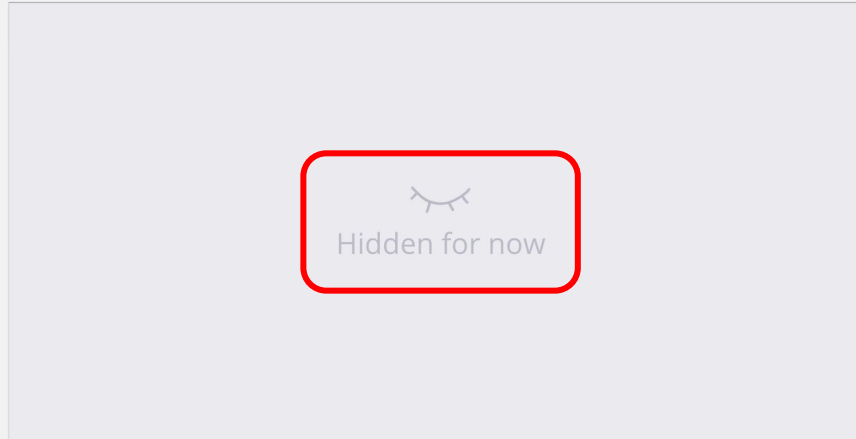**RULE 1**: Each domino should cover two squares of the board.

RULE 2: ...her.

RULE 3: Dominoes are not allowed to cover the "off limits" marker.

Long press to unlock

Student A

Activity 2 yet?

Activity 2

Hidden for now

Day 1 Purposes

Dominoes on 8x8 chess boards

Warm-Up 1	Warm-Up 2	Activity 1
- Miro functionality - Reviewing rules (*uniqueness*)	- New rule with 1 "X" (*divisibility*) - Miro functionality	Generalizations with 2 "X"s

Day 1
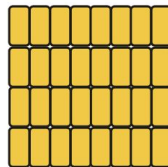
- Miro functionality ✔
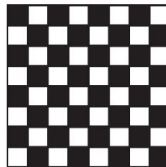- Rules 1 - 2 ✔

Day 1

- Warm-Up 1
- Warm-Up 2
- Activity 1

Directions: Start with an uncovered chessboard. Can you cover the board with dominoes by following these rules?

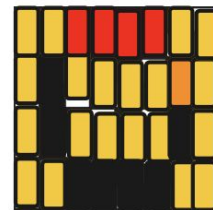**RULE 1**: Each domino should cover two squares of the board.

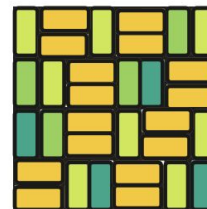**RULE 2**: Dominoes are not allowed to overlap one another.

Follow-Up 1: If your answer is yes...can you find two or three *different* ways of covering the board?

Follow-Up 2: For each of these coverings, how many dominoes did you use? Why does that number make sense?

Student A

Student B

Day 1

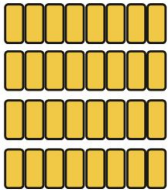- Rule 3 ✔
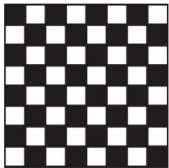- Miro functionality ✔

Day 1
- Warm-Up 1
- Warm-Up 2
- Activity 1

Directions: Start with an uncovered chessboard. Take one of the "off limits" markers (X) and use it to cover the top right corner. Can you cover this board by following these rules?

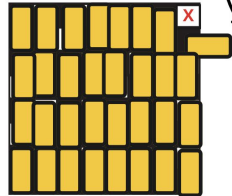**RULE 1**: Each domino should cover two squares of the board.

RULE 2: Dominoes are not allowed to overlap one another.

RULE 3 (new!): Dominoes are not allowed to cover the "off limits" marker.

Follow-Up: Think about your answer to Warm-Up #2. Why cannot you cover the board when one tile is off limits?

Student A

$64 - 1 = 63$

$63 \div 2 = 31 R1$

Day 1

- Generalizations

Day 1

- Warm-Up 1
- Warm-Up 2
- Activity 1

Directions: Start with an uncovered chessboard. Take one of the "off limits" markers (X) and use it to cover two spaces.

Following Rule 1, Rule 2, and Rule 3, which boards can be covered?
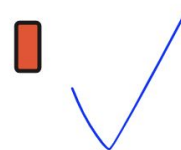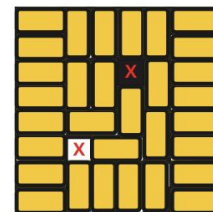
Which boards cannot be covered?

RULE 1: Each domino should cover two squares of the board.

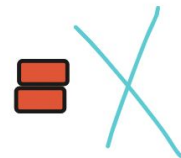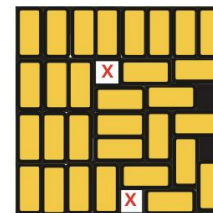**RULE 2**: Dominoes are not allowed to overlap one another.

RULE 3: Dominoes are not allowed to cover the "off limits" marker.

Follow-Up: Do the boards that <u>cannot</u> be covered have anything in common? Explain why the pattern makes sense.

Student C

Student C

Day 1

- **Generalizations** ✔️

Day 1
- Warm-Up 1
- Warm-Up 2
- Activity 1

Directions: Start with an uncovered chessboard. Take one of the "off limits" markers (☒) and use it to cover two spaces.

Following Rule 1, Rule 2, and Rule 3, which boards can be covered?

Which boards cannot be covered?

RULE 1: Each domino should cover two squares of the board.

RULE 2: Dominoes are not allowed to overlap one another.

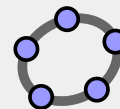**RULE 3**: Dominoes are not allowed to cover the "off limits" marker.

Follow-Up: Do the boards that _cannot_ be covered have anything in common? Explain why the pattern makes sense.

Student B

Student B

Day 1

SCAN ME

Day 1

- Warm-Up 1
- Warm-Up 2
- Activity 1

Directions: Start with an uncovered chessboard. Take one of the "off limits" markers (X) and use it to cover two spaces.

Following Rule 1, Rule 2, and Rule 3, which boards can be covered?

Which boards cannot be covered?

RULE 1: Each domino should cover two squares of the board.

RULE 2: Dominoes are not allowed to overlap one another.

RULE 3: Dominoes are not allowed to cover the "off limits" marker.

Follow-Up: Do the boards that <u>cannot</u> be covered have anything in common? Explain why the pattern makes sense.

Chessboard   Yellow Dominoes   Orange Dominoes

www.geogebra.org

Dominoes on a Chessboard

Dominoes on a Chessboard

Day 2 Purposes

Trominoes

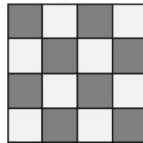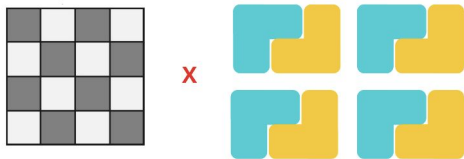| Warm-Up 3	Activity 2	Activity 3
● Miro functionality ● Reviewing rules (*divisibility*) ● Review Day 1	● Rules with 1 "X" with 4x4 board (*base case*)	● Rules with 1 "X" with 8x8 board (*induction*) ● GeoGebra functionality

Day 2

Day 2

- Warm-Up 3
- Activity 2
- Activity 3

Directions: Start with an uncovered 4x4 board. Can you explain why it is _impossible_ to cover the board with L-shaped "trominoes" if you must follow these rules?

RULE 1: Each tromino should cover three squares of the board.

RULE 2: Trominoes are not allowed to overlap.

_Hint_: How many spaces would be covered by 1 tromino? 2 trominos? 3 trominos? 5 trominos?

Day 2

- **Rules 1 - 3** ✔️

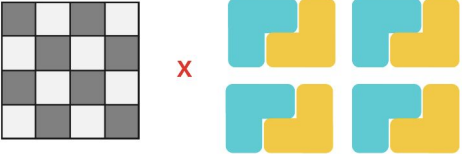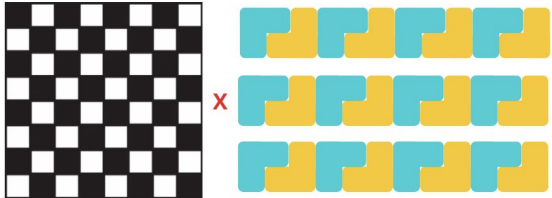| Day 2 |
|---|
| - Warm-Up 3 |
| - **Activity 2** |
| - Activity 3 |

Directions: Start with an uncovered 4x4 board. Take one of the "off limits" markers (**X**) and use it to cover one space. Try to cover the board by following these rules:

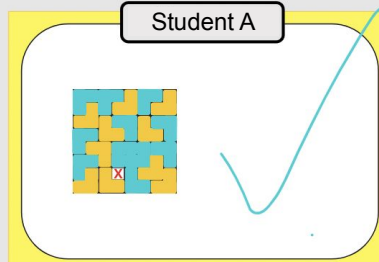**RULE 1**: Each tromino should cover three squares of the board.

RULE 2: Trominoes are not allowed to overlap.

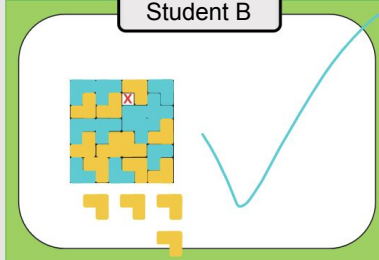**RULE 3 (new!)**: Trominoes are not allowed to cover the "off limits" marker.

Student A
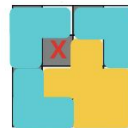
Student B
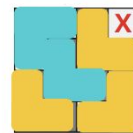
Student C

Day 2

- **Rules 1 - 3** ✔️

SCAN ME

Day 2
• Warm-Up 3
• Activity 2
• Activity 3

Directions: Start with an uncovered 4x4 board. Take one of the "off limits" markers (❌) and use it to cover one space. Try to cover the board by following these rules:

RULE 1: Each tromino should cover three squares of the board.

RULE 2: Trominoes are not allowed to overlap.

RULE 3 (new!): Trominoes are not allowed to cover the "off limits" marker.

Chessboard   Trominoes

www.geogebra.org

Trominoes on a 4x4 Chessboard

Trominoes on a 4x4 Chessboard

Day 2

- **Generalizations** ❌

Directions: Start with an uncovered 8x8 board. Take one of the "off limits" markers (❌) and use it to cover one space. Try to cover the board by following these rules:

RULE 1: Each tromino should cover three squares of the board.

RULE 2: Trominoes are not allowed to overlap.

RULE 3 (new!): Trominoes are not allowed to cover the "off limits" marker.
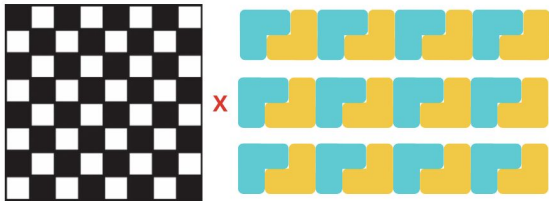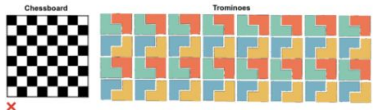
Follow-Up: Is there some "Divide and Conquer" strategy that we could use to make this task easier to complete?
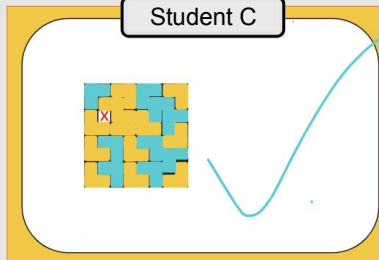
Student A

Student B

Student C

Divide and Conquer Strategy

Step 1:
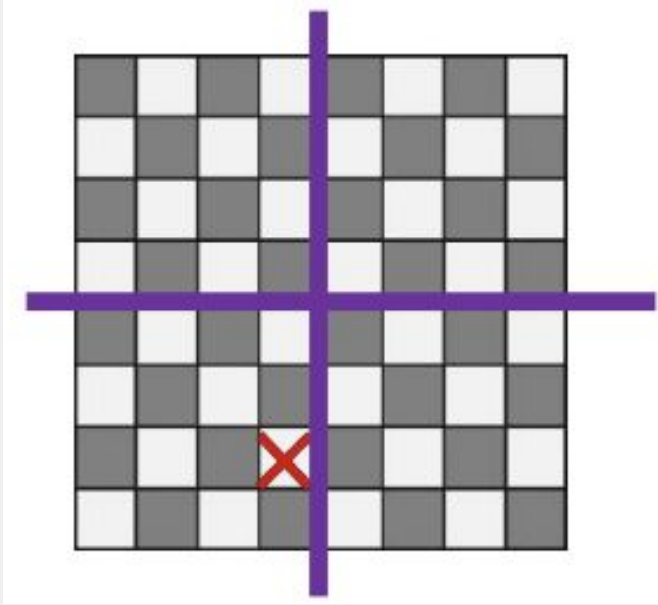
Step 2:
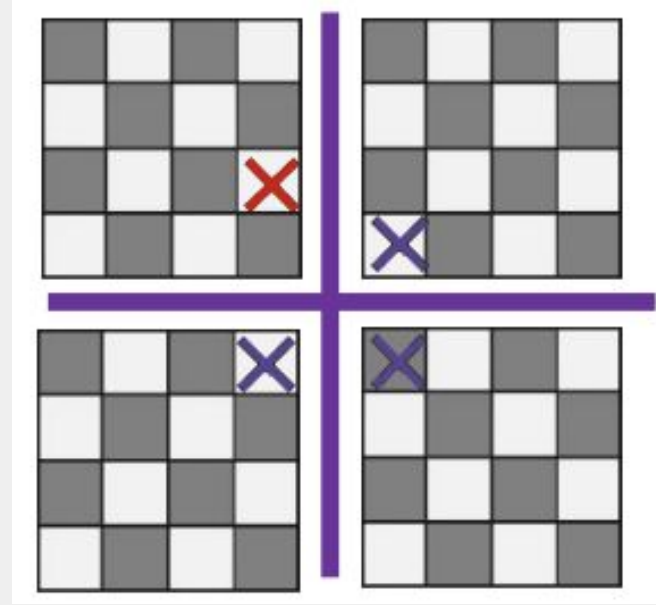
Divide and Conquer Strategy

Step 3:

Step 4:

Day 2

SCAN ME

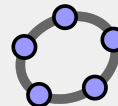## Day 2

- Warm-Up 3
- Activity 2
- Activity 3

Directions: Start with an uncovered 8x8 board. Take one of the "off limits" markers (**X**) and use it to cover one space. Try to cover the board by following these rules:

RULE 1: Each tromino should cover three squares of the board.

RULE 2: Trominoes are not allowed to overlap.

RULE 3 (new!): Trominoes are not allowed to cover the "off limits" marker.

x

Follow-Up: Is there some "Divide and Conquer" strategy that we could use to make this task easier to complete?

Chessboard   Trominoes

x

www.geogebra.org

Trominoes on an 8x8 Chessboard

Trominoes on an 8x8 Chessboard

Day 2
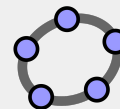
- GeoGebra functionality ✔️

Day 2

- Warm-Up 3
- Activity 2
- Activity 3 - 4x

Trominoes on an 8x8 Chessboard with 4 X's

Author: Christina Durón

Can you arrange the red, orange, blue, and green trominoes to cover the 8x8 chessboard, without any trominoes overlapping?

Chessboard

Trominoes

Day 2

Day 2

- Warm-Up 3
- Activity 2
- Activity 3
- Extension

Anatomy of GeoGebra

SCAN ME
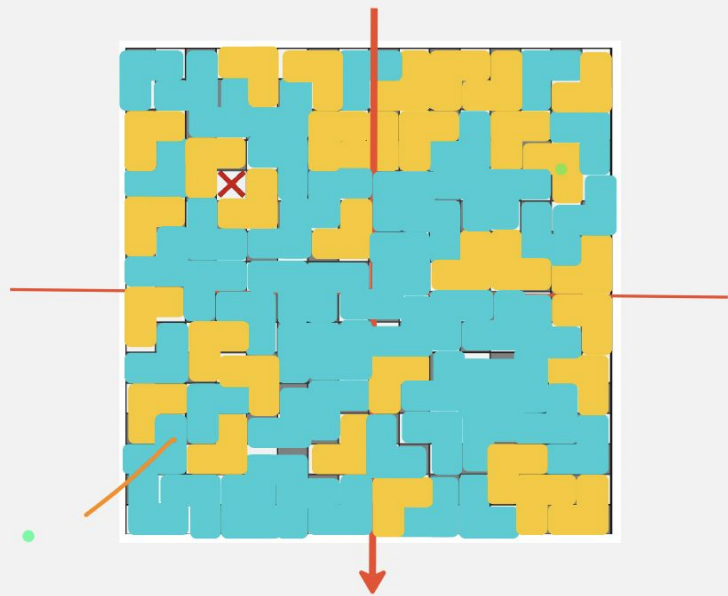
Algebra

A = (−54.68, −
B = (18.43, −2
C = (27.05, 15
D = (44.72, 15
E = (26.57, 33.
F = (44.97, 33.
G = (36.25, 34
H = (53.92, 34
I = (36.49, 14.74)
J = (54.41, 14.74)
A_1 = (−47.39, −57.04)
B_1 = (25.72, −57.04)
C_1 = (27.17, −21.94)
D_1 = (44.85, −21.94)
E_1 = (26.69, −3.54)
F_1 = (45.09, −3.54)
G_1 = (36.37, −3.06)
H_1 = (54.04, −3.06)
I_1 = (36.61, −22.67)
J_1 = (54.53, −22.67)
A_2 = (−18.58, −21.7)
B_2 = (54.53, −21.7)
C_2 = (56.22, 15.1)
D_2 = (73.9, 15.1)
E_2 = (55.74, 33.5)
F_2 = (74.14, 33.5)
G_2 = (65.42, 33.98)
H_2 = (83.1, 33.98)

New
Open ...   ⌘ O
Open Recent   ▸
Save   ⌘ S
Save As...
Share...
Export   ▸
Print Preview   ⌘ P
Close   ⌘ W

Web page (html) ...   ⇧ ⌘ W
Graphics View as Picture (png, eps) ...   ⇧ ⌘ U
Graphics View as Animated GIF ...
Graphics View to Clipboard   ⇧ ⌘ C
Graphics View as PSTricks ...   ⇧ ⌘ T
Graphics View as PGF/TikZ ...
Graphics View as Asymptote ...
Collada...
Collada (html)...

Trominoes

Input:

ABC   a=2

Variations

- **Nicole Sullivant** (Grades 7-8)

Domino Covering Challenge

Question: What conditions determine if these oddly-shaped boards can be covered by dominos?

Can you arrange the yellow and orange dominoes in a way that covers the shaped boards by following these rules?

Rule 1: Each domino should cover 2 squares
Rule 2: Dominoes should not overlap

Question: What conditions determine if these oddly-shaped boards can be covered by dominos?

Domino Covering: Proofs

Prove the following propositions:
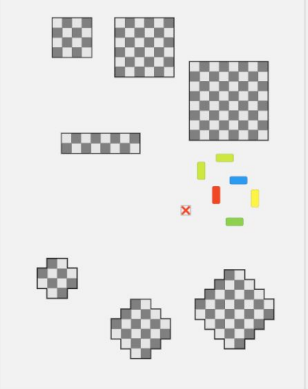
1. If a rectangular board has an even number of squares, then it must have an equal number of light squares and dark squares.

2. It is always possible to cut an L-shaped board into two rectangular regions, where one of the region has an even number of squares.

3. If an L-shaped board has an even number of squares, then it must have an equal number of light squares and dark squares.

SCAN ME

Variations

- **Ibrahim Fatkullin** (Grades 9-12)

Thank you for your time!

Miro Tutorial

GeoGebra Tutorial

Please do not hesitate to reach out to us!

Christina Durón   christina.duron@pepperdine.edu

John Peca-Medlin   johnpeca@math.arizona.edu